Theory of Plasma Contactors for Electrodynamic Tethered Satellite Systems

D. E. Parks* and I. Katz†
S-CUBED, La Jolla, California

Recent data from ground and space experiments indicate that plasma releases from an object dramatically reduce the sheath impedance between the object and the ambient plasma surrounding it. Available data are in qualitative accord with the theory developed herein to quantify the flow of current in the sheath. Electron transport in the theory is based on a fluid model of a collisionless plasma with an effective collision frequency comparable to frequencies of plasma oscillations. The theory leads to low effective impedances varying inversely with the square root of the injected plasma density. To support such a low-impedance mode of operation, using an argon plasma source for example, requires $I_P \sim I_e/30$; that is, only one argon ion must be injected for each thirty electrons extracted from the ambient plasma. The required plasma flow rates are quite low; to extract one ampere of electron current requires a mass flow rate of about one gram of argon per day.

Nomenclature

A	= atomic weight
a	= radius of collector of electrons
B	= the geomagnetic field
c	= speed of light (3×10^{10} cm/s)
e	= proton's charge (4.8×10^{-10} esu)
I	= total collected current
I_1	= net rate of escape of ions from vicinity of collector times proton's charge
I_e	= electron collection rate times proton's charge
I_p	= ion generation rate times proton's charge
j	= net current density in plasma
j_e	= random electron current density
j_i	= random ion current density
M	= ion mass
m	= mass of an electron
$\dot{m}$	= mass flow rate in g/s
m_H	= mass of proton
$n \equiv n(r)$	= density of ions at position r
p_c	= electron pressure
p_i	= ion pressure
q	= 1.6×10^{-19} C
R	= $r_o(n_o/n_{amb})^{1/2}$ = effective collection radius
r	= position coordinates of an ion
r_o	= distance from a center of symmetry at which plasma source density is n_o
t	= time
$V = V(r)$	= mean velocity of ions at position r
V_e	= electron thermal velocity
V_i	= ion velocity at matching radius R
Z	= plasma impedance
α	= ν/ω_p
η	= electrical resistivity of plasma
η_{amb}	= ambient plasma density
θ	= electron temperature in energy units
ν	= effective ion/electron collision frequency
ϕ	= electric potential
ϕ_L	= collector potential
ω_p	= $(4\pi ne^2/m)^{1/2}$ = plasma frequency

Received Jan. 27, 1986; revision received June 19, 1986.

*Senior Research Scientist.

†Program Manager.

Introduction

THE electrodynamic tethered satellite system (TSS) requires the ejection of electrons from the Shuttle at one end of the system and the collection of a compensating current by the satellite at the other end. While the simplest concept is to collect electrons on the subsatellite and to collect a corresponding number of positive ions on the Shuttle Orbiter, the ion collection by the Orbiter is acknowledged to be inadequate to support the desired levels of current. The baseline configuration has an electron gun mounted on the Shuttle. To obtain ampere-sized currents, assuming a perveance of 6×10^{-6} A/v$^{3/2}$, requires thousands of volts across the gun. This voltage drop corresponds to an effective emission impedance of thousands of ohms. An alternative method of emitting electrons from the Shuttle is to create a high-density plasma in the vicinity of the Shuttle. The calculations presented here show that sheath impedances are dramatically reduced by the use of hollow-cathode plasma sources.

The passive collection of ampere-level electron currents by the tethered satellite is simple in concept; however, there is also a substantial sheath impedance associated with the flow of charge between the tethered satellite and the ambient space plasma environment. Theory[1] predicts that the extraction of amperes of electron current by a sphere of 1.5-m diameter requires a potential of kilovolts. This high-impedance collection is in substantial accord with the results of the Plasma Interaction Flight Experiment (PIX),[2] which collected only a few milliamperes of current with a kilovolt bias on a 2000-cm^2 solar panel. Both theory and flight data demonstrate clearly the need to increase the current flow between the TSS and the space plasma.

One way to collect more electrons is to increase the diameter of the tethered satellite, but this is impractical for TSS-1, the first TSS mission. Another way is to increase the plasma current in the vicinity of the subsatellite. This can be done by mounting a plasma source, such as a hollow cathode, on the subsatellite. The SEPAC electron beam experiments conducted on Space Lab 1 indicate plasma sources to be an effective means for neutralizing beam currents and controlling spacecraft potentials.[3] When a plasma cloud is ejected along with a 5-keV, 0.3-A electron beam, the spectrum of returning electrons was confined to energies below the beam energy, and the orbiter potential was clamped to a small value on the order of 1 v. When the plasma jet was not active, however, the electron energy spectrum developed a peak at 1.1 keV and there were significant fluxes of electrons above the primary beam energy. The SEPAC experiments provide clear evidence of the low-impedance neutralization of a high-current electron beam by a plasma plume.

The next section gives a brief review of some properties of hollow-cathode sources. The following sections develop a model for estimating the impedance to current flow across the plasma produced by the hollow cathode source, and determines the rate of plasma production required to support a low-impedance mode of current collection.

Hollow-Cathode Plasma Sources

One device for generating a contactor plasma is the hollow cathode. Hollow-cathode devices have played a prominent role in space applications, especially in the development of ion thrusters for solar electric propulsion systems. Ion beams ejected by ion accelerators were charge- and current-neutralized by electron currents flowing from the cathode through the plasma generated by the hollow cathode. The concept of the hollow cathode as a beam neutralizer was successfully incorporated into the SERT II satellite and performed in space flight tests in the manner expected on the basis of tests conducted in high-vacuum laboratory facilities.[4]

The hollow cathode is a compact, low-impedance device. The simplified features of one such device are indicated schematically in Fig. 1.[5]

A neutral gas, such as mercury or argon, flows into the cathode chamber where it is ionized by field-accelerated electrons emitted from the coated insert or chamber walls through thermionic or other processes. The keeper electrode assists in initiating and stabilizing the electrical discharge. With these devices, nearly complete ionization of the neutral gas can be achieved, the resulting plasma flowing through the orifice at the net upstream flow rate. Various devices of this type have been operated at mass flow rates ranging from micrograms per second to grams per second with currents ranging from milliamperes to kiloamperes.[5-7] For applications to the electrodynamic tethered satellite system, primary interest attaches to the low flow rate and low current range.

The hollow cathode used in the SERT experiment had a length of about 10 cm, an external diameter of about one-half centimeter and an orifice diameter of about 1 mm. It used Hg as the operating gas. Mercury flow rates of the order of 100 ma equivalent, or less, neutralized beam currents of the order of 250 ma while developing potential differences no greater than a few tens of volts between various vehicle surfaces and the neutralizing plasma. The hollow cathodes employed in the electrodynamic tether experiment may have physical characteristics similar to those used in the SERT test but should be flexible enough to permit the generation of a substantial range of plasma densities near the vehicle.

Experiments show that the properties of the plasma generated by the hollow cathode depend upon whether it operates in its spot or plume mode.[5] More complete ionization, higher plasma densities and electron temperatures, and a lower electrical impedance of the discharge generally characterize the spot mode. The plume mode is characterized by less efficient ionization, a lower plasma density, and a higher electrical impedance to the flow of discharge current than the spot mode. A higher rate of gas flow, shorter cathode to anode distance, and a higher discharge current tend to produce the spot mode. Figure 2 shows an example of a measured discharge voltage current characteristic.[7]

Hollow Cathodes as Electron Emitters

Hollow cathodes have been used as plasma sources in ground test facilities and in space flight tests to neutralize the charge and current of ion beams of solar-powered ion propulsion systems. In the space flight tests electrons were transported long distances from their source along the path of neutralized high-energy ion beams. There have not been experiments that address the question of how effectively electron currents may flow from hollow-cathode sources into the ambient plasma in the absence of an ion beam. Thus, the following conclusions must be regarded as tentative.

Experiments conducted in ground facilities indicate that the plasma, despite long classical collisional mean free paths, appears to behave in a resistive manner.[8-10] Previous calculations of neutralizer plasmas showed that, at least for regions of several centimeters from the cathode orifice, the plasma properties and electron current flow patterns conformed to a fluid model of electron transport.

The basic elements of the model are the steady-state ion continuity and momentum equations

$$\nabla \cdot nV = 0 \tag{1}$$

$$m_i \frac{\mathrm{d}V}{\mathrm{d}t} = e\left(E + \frac{V \times B}{c}\right) - \frac{\nabla p_i}{n} \tag{2}$$

The motion of the ion is influenced by the ambient magnetic field $\boldsymbol{B}$, the ion pressure p_i (both set to zero in previous studies), and the electric field in the quasineutral plasma. Quasineutrality, together with the assumption that electrons issuing from the cathode orifice satisfy the momentum balance equation

$$\nabla p_e + enE = n\eta e j \tag{3}$$

relate the electron pressure

$$p_e = n\theta \tag{4}$$

the electric field, and the net current density $\boldsymbol{j}$. If the plasma is nonresistive, isothermal, and $\eta = 0$, Eq. (3) yields the Boltzmann law

$$n = \exp(e\phi/\theta) \tag{5}$$

relating the density and the electric potential. In general, the plasma resistivity η is related to an effective collision frequency ν by

$$\eta^{-1} = \frac{ne^2}{m}\nu^{-1} \tag{6}$$

where for a sufficiently dense and cold plasma ν is the classical electron/ion collision frequency. If the plasma is not collision-dominated, the randomization of electron velocities may still occur through enhanced levels of fluctuating electric fields

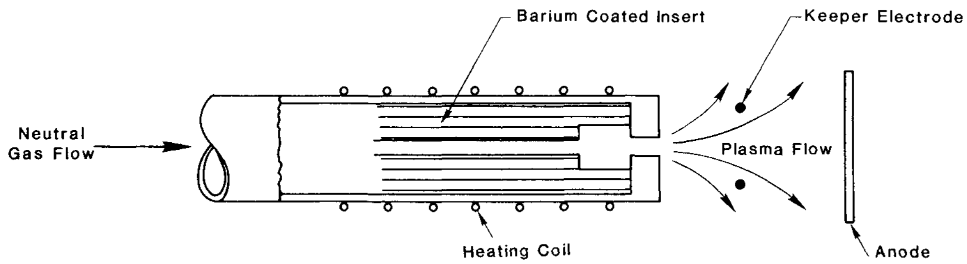

Fig. 1 Schematic diagram of a hollow-cathode configuration.[5]

such as occur in the unstable passage of electron streams through the plasma. These mechanisms are effective in coupling hollow-cathode electrons into the plasma at effective collision frequencies that may be almost as large as the plasma frequency.[8] When augmented by an energy balance equation, two-dimensional calculations predict temperatures and potentials in reasonable agreement with ion engine neutralization data obtained both in the laboratory and in space.

This theory for electron transport can be applied in simplified form to electron emission to the space plasma. Consider a spherical source with its center chosen as the origin of a coordinate system. The plasma is assumed to be isothermal, and its density through space is given by

$$n=\frac{n_o r_o^2}{r^2} \tag{7}$$

to a distance R where $n=n_{\rm amb}$;

$$R=r_o\left(\frac{n_o}{n_{\rm amb}}\right)^{1/2} \tag{8}$$

From Eq. (3)

$$\theta\nabla n-ne\nabla\phi=\eta nej=\eta ne\frac{I}{4\pi r^2} \tag{9}$$

where I is the total current transported to the ambient plasma ($I<0$ for net electron flow outward).

Integrating

$$-\frac{\theta}{e}\ell n\frac{n_o}{n_{\rm amb}}+\phi(r_o)=\frac{I}{4\pi}\int_{r_o}^{R}\eta(r)\frac{dr}{r^2} \tag{10}$$

For a collisionless plasma, it is assumed that

$$\nu=\alpha\omega_p \tag{11}$$

where α is a number less than unity. This form for ν is based upon both theoretical considerations and experimental observation. In a plasma unstable to ion acoustic waves,[11]

$$\nu\simeq\omega_p\left(\frac{W}{n\theta}\right)$$

where W is the energy density contained in saturated electrostatic fluctuations. In a plasma subject to electron/ion two-stream instability,[12]

$$\nu\simeq\omega_p\left(\frac{m}{M}\right)^{1/3}$$

Experiments conducted by Hamberger and Friedman[13] confirm the $n^{1/2}$ dependence of ν contained in Eq. (11) and appear to support the hypothesis that the ion/acoustic and electron/ion two-stream instabilities are the physical mechanisms that lead to values of effective collision frequencies much greater than could be expected on the basis of classical Coulumb collisions. Utilizing Eq. (6), the density given by Eq. (7), and the equation defining ω_p, we obtain

$$\phi_o(r_o)=\theta\ell n\frac{n_o}{n_{\rm amb}}+9\times10^{11}\frac{\alpha I_{\rm amp}}{r_o\omega_p(r_o)}\frac{1}{2}\ell n\frac{n_o}{n_{\rm amb}} \tag{12}$$

with r_o in cm and ω_p in s^{-1}. The resistive contribution to the impedance is

$$Z=9\times10^{11}\frac{\alpha}{r_o\omega_p(r_o)}\frac{1}{2}\ell n\frac{n_o}{n_{\rm amb}}\Omega \tag{13}$$

The hollow cathode,[7] operating in the spot mode at a flow rate of 100 mA equivalent, produced an electron density of about

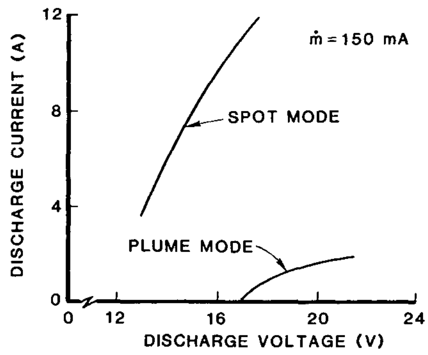

Fig. 2 Discharge voltage-current characteristic.

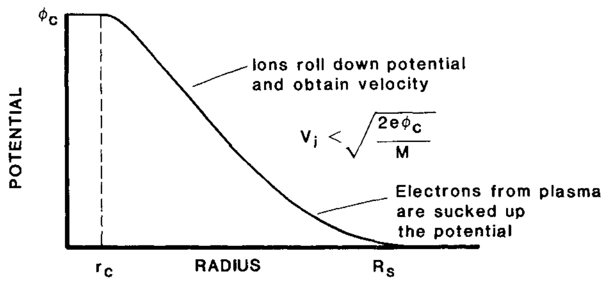

Fig. 3 Potential profile around the plasma source.

10^{12} cm^{-3} at about 1 cm from the orifice. Taking $r_o=1$ cm, $n_o=10^{12}$ cm^{-3}, and $n_{\rm amb}=10^6$ cm^{-3},

$$Z=23\ \Omega$$

for $\alpha=0.1$. Previous studies with this model required $\alpha\sim0.1$; the value $\alpha=1$ probably corresponds to an overestimate. The magnitudes of resistance previously given with $\alpha\sim0.1$ appear consistent with measurements made over the much shorter paths of current conduction involved in laboratory facilities. The value $\alpha\sim0.1$ is also consistent with the magnitude expected from electron/ion two-stream instabilities but does require streaming velocities larger than random electron velocities.

Increasing the density n_o of the injected plasma by two orders of magnitude reduces R by a factor of ten. At densities greater than about 10^{12} cm^{-3} with $\theta<1$ eV, classical scattering should be taken into account, however, since the mean free paths for Coulomb scattering are short (<1 cm at $n=10^{12}$/cm^3).

Hollow Cathodes as Electron Collectors

A sphere whose diameter is much greater than a Debye length will collect an electron current greater than the plasma thermal current into the collector's area by attracting electrons across a space charge-limited sheath. To collect a strongly enhanced current, the potential on the sphere must be much larger than the plasma temperature, i.e., $e\phi\gg\theta$. The current voltage characteristic of such a configuration is well described by the theory of Langmuir and Blodgett.

Most descriptions of the passive collection of electrons by the tethered satellite are based upon space charge-limited sheath theory with some modifications due to magnetic field and presheath effects.[14-16] The theory presented here addresses the changes in the potential structure that occur when a plasma is generated in the vicinity of the sphere. This theory omits the effect of a magnetic field, an omission not totally justified, especially in regions where the electron cyclotron frequency is comparable to or greater than the local plasma frequency.

There is little data on the use of hollow-cathode plasma sources to enhance electron collection. Theoretical considerations, however, support what limited data there are: the effective impedance of an electron collecting probe is greatly reduced by the copious emission of plasma. Even though the theory is incomplete, it identifies the regimes of impedance reduction and defines values of the ion generation rate that will produce substantial changes in the impedance. Increasing the ion generation rate I_p first reduces the voltage drop across the space charge-limited collection sheath, further increases collapse of the space charge sheath, and, when the ion generation rate is increased beyond the electron collection rate I_e, current is transported by the ions.

In fact, the different regimes of current collection may be categorized according to the following inequalities between the ion generation rate I_p and the electron collection rate I_e:

Regime 1:

$$I_p < \sqrt{\frac{m}{M}} I_e$$

Regime 2:

$$\sqrt{\frac{m}{M}} I_e < I_p < I_e$$

Regime 3:

$$I_p > I_e$$

Each of these regimes is considered in the following discussion. For convenience, the collecting sphere is assumed to operate at a constant current.

Regime 1

For a null-ion generation rate, current collection $B=0$ is well understood and requires large voltages to extract current much in excess of the geometrical limit. The effect of generating a small amount of plasma at the sphere is approximately equivalent to that of emitting ions from the anode of a diode. The effect of the electrons created in the ionization process can be ignored if the rate of plasma production is much less than the collected electron current. The plasma ions stream out across the sheath, cancelling a portion of the electron space charge. For planar diodes it has been shown[17] that the maximum ion current density that the sheath can extract is related to the electron current density by

$$j_i = \sqrt{m/M}\, j_e \tag{14}$$

This relation, known as the "Langmuir condition," is also the basic stability condition for a strong plasma double layer. At this ion current, the voltage required to sustain a fixed electron current is reduced by one-third. For nonplanar geometries, this current ratio can be exceeded by factors of the order of two. The resulting small reduction in the sheath voltage is of little importance compared with the dramatic change that occurs when the plasma generation rate increases beyond the j_i of the double-layer stability condition that separates Regimes 1 and 2.

Regime II

Recent calculations of the effect of ionization in electron-collecting sheaths have shown that when the "Langmuir condition" ion current is exceeded, the generated plasma remains quasineutral and the ions expand hydrodynamically.[18] In the limit of the ion generation rate that is large compared with the "Langmuir condition" ion current, and assuming constant temperature, the ion density is determined by the self-consistent motion in the quasineutral field. The resultant description of the potentials and densities is the same as that for hollow-cathode neutralizers used as electron emitters. What is not certain is the magnitude of the electron transport coefficients. The model described in the previous section for electron emission can serve as a first estimate of electron collection from the ambient plasma. The collection area enhancement possible while maintaining isothermal quasineutrality can be obtained by substituting for the total current in terms of the ambient thermal current times an effective collection area, i.e.,

$$I = j_e 4\pi R^2 \tag{15}$$

For a collector radius a of one meter, this estimated collector area exceeds the geometric area by the factor of

$$\frac{R^2}{a^2} = \frac{r_o^2}{a^2} \frac{n_o}{n_{\text{amb}}}$$

Taking $r_o = 1$ cm and $n_o/n_{\text{amb}} = 10^6$,

$$\frac{R^2}{a^2} = 100$$

and by Eq. (13) this enhancement of the collection area is accompanied by an impedance of 23 Ω. Further study is necessary to determine the accuracy of this collection area enhancement.

The ion generation rate required to sustain this lower impedance mode of electron collection can be estimated as follows. At the effective collection radius R defined by Eq. (15), the ion current from the plasma source is

$$I_p = 4\pi R^2 j_i = 4\pi R^2 n_i V_i$$

Since $n_i \simeq n_{\text{amb}}$, at the radius R [it is assumed here that R is the radius where the density of the generated plasma equals that of the ambient plasma; see Eq. (7)]

$$I_p = 4\pi R^2 n_{\text{amb}} V_i = I_e \frac{V_i}{V_e}$$

where V_e is the thermal speed of ambient electrons and V_i is the speed of source ions at the effective collection radius R. As indicated schematically in Fig. 3, the net movement of ions is down the potential hill separating the collector from the ambient plasma.

Neglecting the effect of drag, ions starting from rest would attain the maximum velocity $(2e\phi_c/M)^{1/2}$, so that

$$I_p < I_e (me\phi_c/M\theta)^{1/2}$$

and the bound on the required ion current varies only as the square root of the potential. From Eq. (12) and the following discussion, electron currents near one ampere would correspond to potentials $\phi_c \sim 10$ v. For an argon plasma with $\sqrt{M/m} \sim 300$ and for $\theta \sim 0.1$ eV,

$$I_p \leq \frac{I_e}{30}$$

Regime 3

For ion generation rates in excess of the collected currents, the net electron current is outward from the subsatellite. In this case the ions transport the current, and the effective mobility of the electrons plays little role in determining the plasma potential, provided that the current does not exceed the net rate of escape of ions I_1 from the vicinity of the collector. Thus, assuming the full ionization of the neutral gas flow through the cathode, the required mass flow rate for ion transport is

$$I_1 \leq I_p = \frac{\dot{m}q}{Am_H} = 10^5 \frac{\dot{m}}{A} \text{amps}$$

where $\dot{m}$ is the mass flow rate in g/s of atoms of atomic weight A and m_H is the proton mass. Of course, if the cathode does not float with respect to the collector, the total current through the cathode may exceed I_1, but any current through the cathode/collector/plasma loop does not flow through the tether. It is useful to observe that $I_p \simeq 1$ amps corresponds to a flow rate slightly less than A grams per day. Since high ionization efficiencies are achievable with hollow-cathode sources operating in their spot modes, it is unlikely that such flow rates for the duration of TSS-1 would significantly impact the satellite mass.

Conclusion

For both the electron-emitting and electron-collecting ends of the tethered satellite system, locally generated plasmas eliminate the space charge sheath. The high voltages necessary to transport charge across a space charge sheath makes the sheath regions the highest impedance portions of the tether system. Reducing this impedance by local plasma sources such as hollow cathodes will greatly enhance the effectiveness of a tethered satellite system. The theory is not yet fully developed; e.g., magnetic fields have not been included and the possibility of the formation of double layers has not been examined. The theory does provide an initial framework for understanding how currents flow through the locally generated plasmas. The theory predicts the impedance for electron emission from the orbiter as a function of the ion generation rate I_p, the tether current I, and the ambient plasma density. For electron collection by the subsatellite, the theory predicts three different collection regimes:

1) $I_p < \sqrt{(m/M)}I$, high-impedance space charge-limited collection

2) $\sqrt{(m/M)}I < I_p < I$, resistive quasineutral transport of electrons

3) $I < I_p$, low-impedance ion transport.

Electron emission and all modes of electron collection are well within the capabilities of present-technology hollow-cathode plasma sources. Regime 2 is of primary interest, permitting low-impedance electron collection for low plasma production rates. For argon plasma emitted into the ionosphere in low Earth orbit, the ion production rate I_p required to extract a current I_e from the ambient plasmas satisfies $I_p < I_e/30$.

Acknowledgment

This work was supported by the NASA Lewis Research Center, Cleveland, OH, under Contract NAS3-23881.

References

[1]Al'pert, Y. L., Gurevich, A. V., and Pitaevski, L. P., *Space Physics with Artificial Satellite*, Consultants Bureau, New York, 1965.

[2]Mandell, M. J., Katz, I., Jongeward, G. A., and Roche, J. C., "Computer Simulation of Plasma Electron Collection by PIX-II," AIAA 85-0386, AIAA 23rd Aerospace Sciences Meeting, Reno, NV, 1985 (to appear in *Journal of Spacecraft and Rockets*).

[3]Reasoner, D. L., Burch, J. L., and Obayashi, T., "Analysis of Electron Spectra Produced by SEPAC Plasma Interactions," EOS, *Transactions of the American Geophysical Union*, Vol. 65, 1984, p. 1042.

[4]Jones, S. G., Staskus, J. V., and Byers, D. C., "Preliminary Results of Sert II Spacecraft Potential Measurements Using Hot Wire Emissive Probes," NASA TM-X-2083, 1970.

[5]Csiky, G. A., "Measurements of Some Properties of a Discharge from a Hollow Cathode," NASA Technical Note, NASA-TN-D-4966, Feb. 1969.

[6]Knishnan, M., John, R. G., von Jaskowsky, W. F., and Clark, K. E., "Physical Processes in Hollow Cathodes," *AIAA Journal*, Vol. 15, Sept. 1977, pp. 1217–1223.

[7]Siegfried, D. E. and Wilbur, P. J., "An Investigation of Mercury Hollow Cathode Phenomena," AIAA Paper 78-0705, April 1978.

[8]Parks, D. E., Mandell, M. J., and Katz, I., "Fluid Model of Plasma Outside a Hollow Cathode Neutralizer," *Journal of Spacecraft and Rockets*, Vol. 19, July-Aug. 1982, pp. 354–357.

[9]Katz, I., Cassidy, J. J., Mandell, M. J., Parks, D. E., Schnuelle, G. W., Stannard, P. R., and Steen, P. G., *Additional Application of the NASCAP Code*, Vol. 2, "Ion Thruster Neutralization and Electrostatic Antenna Model," SEPS, NASA CR-165350, Feb. 1981.

[10]Ward, J. W. and King, H. J., "Mercury Hollow Cathode Plasma Bridge Neutralizers," *Journal of Spacecraft*, Vol. 10, 1968, p. 1161.

[11]Sagdeev, R. Z., "The 1976 Oppenheimer Lectures: Critical Problems in Plasma Astrophysics, I. Turbulence and Nonlinear Waves," *Review of Modern Physics*, Vol. 51, 1979.

[12]Bunemann, O., "Dissipation of Currents in Ionized Media," *Physical Review*, Vol. 115, 1959.

[13]Hamberger, S. M. and Friedman, M., "Electrical Conductivity of a Highly Turbulent Plasma," *Physical Review Letters*, Vol. 21, 1968.

[14]Parks, D. R., Katz, I., Jongeward, G. A., and Rotenberg, M., "Electrodynamic Tether Study," S-CUBED Final Report (Draft), SSS-R-85-6883, Sept. 1984.

[15]Grossi, M. and Arnold, D. A., "Engineering Study of the Electrodynamic Tether as a Spaceborne Generator of Electric Power," Smithsonian Astrophysical Observatory, SAO Technical Report, NASA Contract NAS8-35497, June 1984.

[16]Arnold, D. A. and Dobrowolny, M., "Transmission Line Model of the Interaction of a Long Metal Wire with the Ionosphere," *Radio Science*, Vol. 15, 1980, p. 1149.

[17]Langmuir, I., "The Interaction of Electron and Positive Ion Space Charges in Cathode Sheaths," *Physical Review*, Vol. 33, 1929, p. 954.

[18]Cooke, D. L. and Katz, I. "Ionization Induced Instability in an Electron Collecting Sheath," S-CUBED Report, May 1985.